

*January Exam**Cell. Mob. Comm. Sys.*

This formula sheet provides the additional formula that are not given on the exam-provided formula sheet for the final Cellular Mobile Communication Systems module run by Dr. Ahmed Lawey.

SIGNALS

Sinusoidal Signal:

$$s(t) = A \sin(2\pi ft + \theta)$$

$$P_s = \frac{A^2}{2}$$

$$B_s = \frac{1}{T_s}$$

, where P_s is signal power, B_s is signal bandwidth, T_s is symbol time.

LOS PROPAGATION

Path Loss:

$$\begin{aligned} \text{PL} &= \frac{P_t}{P_r} \\ &= P_t - P_r \quad (\text{in dB}) \end{aligned}$$

Free Space Path Loss:

$$\begin{aligned} L_{\text{free}} &= \left(\frac{4\pi d}{\lambda} \right)^2 \\ &= -20 \log_{10} \left(\frac{\lambda}{4\pi d} \right) \quad (\text{in dB}) \end{aligned}$$

Friis' Equation:

$$\begin{aligned} P_r &= \frac{P_t G_t G_r}{L_{\text{free}} L} \\ &= P_t + G_t + G_r - L_{\text{free}} - L_{\text{coupling}} \quad (\text{in dB}) \end{aligned}$$

Power at Reference Distance:

$$\begin{aligned} P_r &= P_{r,d_{\text{ref}}} \left(\frac{d_{\text{ref}}}{d} \right)^v \\ &= P_{r,d_{\text{ref}}} + 10v \log_{10} \left(\frac{d_{\text{ref}}}{d} \right) \quad (\text{in dB}) \end{aligned}$$

Note that $v = 2$ for LOS, $2 < v < 6$ for NLOS.

NLOS PROPAGATION

Okumura-Hata Path Loss Exponent:

$$v = \frac{P_{\text{loss}} - P_{\text{loss,ref}}}{10 [\log_{10}(d) - \log_{10}(d_{\text{ref}})]} \quad (\text{in dB})$$

Okumura-Hata Path Loss at Ref. Distance:

$$P_{\text{loss,ref}} = 10 \log_{10} \left[\frac{(4\pi d_{\text{ref}})^2}{\lambda^2} \right] \quad (\text{in dB})$$

SHORT-TERM FADING

Outage Probability:

$$\begin{aligned} P_{\text{outage}} &= 1 - \exp \left(-\frac{P_{\text{threshold}}}{P_{\text{average}}} \right) \\ &= 1 - \exp \left(-\frac{SNR_{th}}{SNR_o} \right) \end{aligned}$$

Mean Channel Delay:

$$\begin{aligned} \langle \tau \rangle &= \frac{\sum_{i=1}^N p_i \tau_i}{\sum_{i=1}^N p_i} \\ \langle \tau^2 \rangle &= \frac{\sum_{i=1}^N p_i \tau_i^2}{\sum_{i=1}^N p_i} \end{aligned}$$

RMS Delay Spread:

$$\sigma_d = \sqrt{\langle \tau^2 \rangle - \langle \tau \rangle^2}$$

Channel Bandwidth:

$$\begin{aligned} B_c &= \frac{1}{5\sigma_d} \\ B_c > B_s &: \text{flat fading} \\ B_s > B_c &: \text{frequency selective} \end{aligned}$$

Rician PDF:

$$\begin{aligned} K &= 10 \log_{10} \left(\frac{A_0^2}{2\sigma^2} \right) \\ &= 10 \log_{10} \left(\frac{\text{LOS power}}{\text{NLOS power}} \right) \end{aligned}$$

LONG-TERM FADING

Lognormal Received Power:

$$P_r = P_{r,d_{ref}} + 10v \log_{10} \left(\frac{d_{ref}}{d} \right) + Y_g$$

$$\sigma = \frac{\sigma_{dB} \ln(10)}{10} \quad (\text{for lognormal distribution})$$

Note that Y_g is a zero-mean random gaussian vale with σ_{dB} standard deviation. σ is for the lognormal distribution, see exam formula sheet.

FREQUENCY SPREADING

Freq. Shift:

$$f_d = f_0 \frac{v}{c}$$

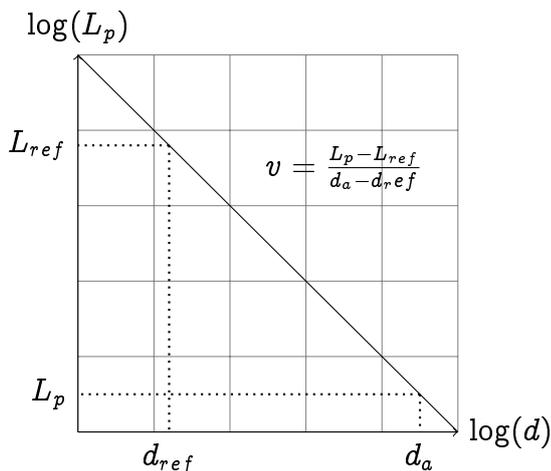
$$f_{in} = f_0 + f_d \cos(\theta_i)$$

$$T_s < T_c; B_s > f_d : \text{slow fading (good)}$$

$$T_c < T_s; f_d > B_s : \text{fast fading (bad)}$$

UNDERSTANDING v CALCULATION

Plotting the path loss (L_p) against distance (d), you can see that the curve is inversely exponential. When the logarithm of both sides is taken, the graph becomes inversely linear. Given a reference distance (d_{ref}) and actual distance (d_a), we can calculate the gradient of the line. THIS GRADIENT IS THE LOSS EXPONENT v , as v is the rate that L_p decreases as distance increases.



CELLULAR SYSTEMS

Distance btwn. neighbouring cells: $\sqrt{3}r$
Distance btwn. co-channel cells (u_1, v_1), (u_2, v_2):

$$D = \sqrt{i^2 + j^2 + ij\sqrt{3}r} \quad (\text{where } i, j \text{ are axis movements along cell})$$

$$N_c = i^2 + j^2 + ij$$

Freq. Re-use Factor: $q = \frac{D}{R}$
Co-Channel Interference (general):

$$SIR = \frac{1}{6} (3N_c)^{v/2}$$

$$= \frac{1}{6} \left(\frac{D}{R} \right)^v$$

Co-Channel Interference (at edge of cell):

$$SIR = \frac{1}{6(q-1)^{-v}}$$

Note a larger cluster size results in a higher SIR, but a lower system capacity (less users). For directional antennas, 120° has 2 interferers, 60° has 1 interferer.

Cellular System Capacity:

$$\text{System Capacity} = \text{No. of Clusters in area} \times \text{No. of Channels in Clusters}$$

Power at (unsplit) Cell Boundary:

$$P_u = P_t R^{-v}$$

Power at (split) Cell Boundary:

$$P_{su} = P_{st} (R/2)^{-v}$$

$$P_{st} = P_t / 2^v$$

The previous equation assumes that the minicell has a radius half that of the unsplit cell. Handling split cells introduces complexity in handover procedures for a decrease in Tx power, decrease in users per cell & higher re-use of channels (thus higher system capacity).

Fading Margin Calculation:

$$M = 10 \log_{10} \frac{P_o}{P_{th}}$$

Fade margin is a margin to allow sufficient coverage to accomodate expected fading losses.

FADING MITIGATION VIA DIVERSITY

Outage Probability for M receivers:

$$P_{\text{out}} = [1 - e^{P_{th}/P_o}]^M$$

WIRELESS SYSTEM CAPACITY

Maximum Capacity for AWGN Channel:

$$\begin{aligned} C_{\text{awgn}} &= W \log_2(1 + SNR) \\ &= W \log_2\left(1 + \frac{\bar{P}}{N_0 W}\right) \\ &= \frac{\bar{P}}{N_0} \log_2(e) \quad (\text{SNR} \ll 1) \\ &= W \log_2\left(\frac{\bar{P}}{N_0 W}\right) \quad (\text{SNR} \gg 1) \end{aligned}$$

Rate of Reliable Comm. at Edge of Cell:

$$R_p = \rho W \log_2\left(1 + \frac{SNR}{\rho + f(\rho)SNR}\right)$$

Users in narrowband systems ($\rho < 1$) have high SINR but low fraction of system bandwidth. Users in wideband systems ($\rho = 1$) have low SINR but high fraction of system bandwidth.

Uplink Capacity Region Bounds (maximum achievable capacity for two users):

$$\begin{aligned} R_1 &< \log_2\left(1 + \frac{P_1}{N_0}\right) \\ R_2 &< \log_2\left(1 + \frac{P_2}{N_0}\right) \\ R_1 + R_2 &< \log_2\left(1 + \frac{P_1 + P_2}{N_0}\right) \end{aligned}$$

Both users cannot transmit at the same time at maximum capacity.

SIC Uplink Maximum Data Rates:

$$\begin{aligned} R_2 &= \log_2\left(1 + \frac{P_2}{P_1 + N_0}\right) \\ R_1 &= \log_2\left(1 + \frac{P_1}{N_0}\right) \end{aligned}$$

Uplink SIC scheme can meet the single user data rate bound for U1 and a non-zero rate for U2.

Orthogonal Uplink Maximum Data Rates:

$$\begin{aligned} R_1 &= \alpha \log_2\left(1 + \frac{P_1}{\alpha N_0}\right) \\ R_2 &= (1 - \alpha) \log_2\left(1 + \frac{P_2}{(1 - \alpha)N_0}\right) \end{aligned}$$

Orthogonal uplink schemes only meet the capacity bounds at the point $\alpha = P_1/(P_1 + P_2)$. CDMA uplink schemes cannot meet the capacity bounds.

Downlink Capacity Region Bounds:

$$\begin{aligned} R_1 &< \log_2\left(1 + \frac{P_1 h_1^2}{N_0}\right) \\ R_2 &< \log_2\left(1 + \frac{P_2 h_2^2}{N_0}\right) \end{aligned}$$

For downlink, each user's power restrictions are separate from other users.

Orthogonal Downlink Maximum Data Rates:

$$\begin{aligned} R_1 &= \alpha \log_2\left(1 + \frac{P_1 h_1^2}{\alpha N_0}\right) \\ R_2 &= (1 - \alpha) \log_2\left(1 + \frac{P_2 h_2^2}{(1 - \alpha)N_0}\right) \end{aligned}$$

Orthogonal coding at the transmitter is beaten by superposition coding at the transmitter. Superposition coding transmits both signals together at once, where receivers treat the other as noise (SIC).

QUESTION PROCEDURE

For calculating LOS received power:

1. Calculate L_{free}
2. Convert powers to dB.
3. Calculate received power in dB
4. Convert to linear received power

For calculating path loss exponent (ν):

1. Calculate LOS L_{free} for reference distance
2. Calculate NLOS L_p for full distance
3. Put into equation for ν (change in path loss against change in distance)